Phase-field model for dielectric breakdown in solids

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Using an analogy between dielectric breakdown and fracture of solids, this paper develops a phase field model for the electric damage initiation and propagation in dielectric solids during breakdown. Instead of explicitly tracing the growth of a conductive channel, the model introduces a continuous phase field to characterize the degree of damage, and the conductive channel is represented by a localized region of fully damaged material. Similar as in the classic theory of fracture mechanics, an energetic criterion is taken: the conductive channel will grow only if the electrostatic energy released per unit length of the channel is greater than that dissipated through damage. Such an approach circumvents the detailed analysis on the complex microscopic processes near the tip of a conductive channel, and provides a means of quantitatively predicting breakdown phenomena in materials, composites, and devices. This model is implemented into a finite-element code and several numerical examples are solved. With randomly distributed defects, the model recovers the inverse power relation between breakdown strength and sample thickness. Finally, the effect of the layered structure in a breakdown-resistant laminate is demonstrated through a numerical example.

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I. INTRODUCTION

Subject to a sufficiently strong electric field, a dielectric solid will dramatically increase its conductivity, resulting in the failure of electric insulation. This phenomenon is commonly known as dielectric breakdown. The dielectric strength, namely the critical electric field of breakdown, is one of the key parameters that limit the performance, safety, and energy density of electric components [1, 2]. However, even in well controlled experiments, the measured critical fields usually carry large statistical scattering [3], and the values are often several orders of magnitude lower than the intrinsic strengths predicted by microscopic models of breakdown [4, 5]. This difference has been attributed mainly to the existence of microscopic flaws, such as cracks, lattice defects, particulate micro-contamination, and localized interface imperfections [6]. Instead of a material constant invariant from sample to sample, the breakdown strength is found to be dependent on the size and distribution of defects, specimen size, ambient conditions, and electrode configurations [7].
Unlike gases or liquids, a dielectric solid usually exhibits a trail of permanent damage after breakdown, often in the form of a tubular conductive channel [8, 9]. The major portion of the material outside the channel remains undamaged. Such a localized damage process is analogous to the fracture of brittle solids, in which irreversible damage only takes place within the thin layers adjacent to crack surfaces. The similarity between dielectric breakdown and fracture was realized as early as in 1927 [10].

Following a Griffith-type energy balance argument, the criterion for the propagation of a conductive crack is derived and used to model dielectric breakdown [11]. Considering the one-dimensional nature of a conductive channel formed at breakdown, later models suggest that the property of interest should be the energy needed to create per unit length of the channel [12], which will be referred to as the breakdown energy and denoted by $\Gamma$ hereafter (in analogy with the fracture energy of a crack). Furthermore, the path-independent surface integration around the tip of a thin conductive channel (in analogy with the J-integral of fracture) is identified and shown to be equal to the breakdown energy at the critical point [13].

Besides theoretical development, there are very few computational methods that quantitatively study the detailed processes of dielectric breakdown, whilst numerous models are well established for fracture, such as the cohesive zone model [14], the extended finite element method [15], and the phase-field approach [16].

In past decades, the phase-field method has become a versatile tool for modeling phase transition and microstructure evolution in various material systems [17-20]. By representing discrete identities (e.g. a crack) by the abrupt but continuous change of a field variable, phase-field models avoid the hassle of tracing the motion of individual identities. This paper aims at developing a phase-field model to study the damage initiation and propagation associated with the dielectric breakdown processes of solids. Inspired by the phase-field model of brittle fracture [16, 21-26], we represent the level of damage by a continuous field, and model the breakdown energy as the surface energy of a tubular conductive channel. This phase-field model is further implemented into a finite-element code, so that the quantitative study on the effects of defects, material microstructures, complex specimen geometries, and loading conditions may be carried out.

II. THE MODEL

A. Phase field and the energy functional

Following Suo [12], we assume the energetic criterion for the propagation of a tubular conductive channel: associated with the extension of the channel by unit length, the release in the conservative
energy of the system, including the electrostatic energy stored in the material and the space and the potential of any external sources, equals the work needed (or energy dissipated) to create unit length of the channel. As the damage evolution and charge redistribution are usually much slower than the speed of light, the energy emission in the form of electromagnetic wave is neglected. We assume the system to be in electrostatic equilibrium at all times of consideration. The energy dissipation through electric conduction, on the other hand, is considered a part of the breakdown energy. Further extension of the model by considering charge conduction is possible but beyond the scope of the current paper. The following discussion will elucidate different packages of energy in the system.

![FIG 1. Sketch of a dielectric medium under applied electrostatic field.](image)

Consider a dielectric medium occupying spatial domain \( \Omega \), as sketched in Fig. 1. The electric field is applied through several conductive electrodes, on which the electric potential \( \phi \) is known. For ease of description, it is assumed that the electrodes are perfectly shielded and there is no leakage of electric field outside \( \Omega \). A material particle responds to the local electric field \( E \) by electric displacement \( D \). For simplicity, we assume the material to be linear dielectric with permittivity \( \varepsilon \) prior to damage. The electrostatic energy stored per unit volume of the material is then given by

\[
W_{es}(D) = \frac{1}{2\varepsilon} D_i D_i ,
\]

in which the repeated index indicates a summation. In equilibrium, the electric field \( E_i = -\partial \phi / \partial x_i \) is related to the electric displacement by \( E_i = \partial W_{es} / \partial D_i \). In the absence of material damage, the total potential energy of the system, including the electrostatic energy in the bulk and the potential of the external sources,
\[ \Pi = \int_{\Omega} W_{es} dV - \int_{\partial \Omega_\phi} \phi \omega dA \]  

(2)
is conservative. The surface integration in Eq. (2) is carried over the area covered by electrodes, \( \partial \Omega_\phi \), and \( \omega \) is the local charge density on the surface. To use the electric potential as the main field variable, we introduce the complementary electrostatic energy function through the Legendre transform

\[ \hat{W}_{es}(E) = W_{es}(D) - E_i D_i = -\frac{\varepsilon}{2} E_i E_i. \]  

(3)
The system potential energy is thus the volume integral of \( \hat{W}_{es} \) only.

Inspired by the phase-field models of brittle fracture [16, 22], we use a scalar phase field \( s(x,t) \) to characterize the state of damage of the material. \( s \) varies from 0 to 1, with \( s = 1 \) indicating the virgin state and \( s = 0 \) the fully damaged state. The degradation of the material needs to be reflected on its dielectric property. In reality, the fully damaged material inside the tubular channel loses its insulating capability and becomes conductive. Here for simplicity, we use a dielectric phase with very high permittivity to mimic the behavior of the conductive phase. The electric work done by the electric field through conducting a current is replaced by the electrostatic energy release through a large electric displacement. Mathematically, we write the permittivity as a function of the phase-field variable \( s \):

\[ \varepsilon(s) = \frac{\varepsilon^0}{f(s) + \eta}. \]  

(4)
Here, \( \varepsilon^0 \) is the permittivity of the intact material, and \( f \) is an interpolation function between \( f(0) = 0 \) and \( f(1) = 1 \). In order not to affect the thermodynamics of the homogeneous phases, it is commonly required that \( f \) has vanishing derivatives in each pure phase: \( f'(0) = f'(1) = 0 \). A practical choice of \( f \) is \( f(s) = 4s^3 - 3s^4 \) [16], and is used throughout this paper. \( \eta \) is a small number added for numerical calculation. In the following examples, we set \( \eta = 10^{-3} \), corresponding to a permittivity in the damaged phase 1000 times that in the intact phase.

Under a nonzero electric field, the damage-dependent complementary electrostatic energy (3) minimizes at the fully damaged state, \( s = 0 \). To have coexisting damaged and undamaged phases, we further introduce an energy function.
\[ W_d(s) = \frac{\Gamma}{l^2} [1 - f(s)]. \]  

(5)

As will be shown in the following section, \( \Gamma \) is related to the breakdown energy, and \( l \) is an intrinsic length scale characterizing the thickness of the conductive channel. \( \Gamma/l^2 \) is the critical electrostatic energy density to initiate local damage. This energy function can be imagined as a buffer that artificially stores the energy consumption during the formation of conductive channel, i.e. the material damage process. Such an approach yields a thermodynamically reversible damage zone, which heals upon electric unloading. To avoid this complication, special numerical schemes corresponding to an asymmetric kinetic law can be taken, just like in the phase-field modeling of brittle fracture [22]. The damage energy (5), together with the electrostatic energy (3), enables the separation of the damaged and intact phases whenever it is energetically favorable. Just like most phase-field models, the diffuse interface between the damaged and intact phases is enabled through a gradient energy term,

\[ W_i(\nabla s) = \frac{\Gamma}{4} \frac{\partial s}{\partial x_i} \frac{\partial s}{\partial x_i}, \]  

(6)

which penalizes a sharp transition between dissimilar phases.

The free-energy per unit volume of the material now consists of the contribution from the electrostatics, the damage, and the interface: \( W(E, s, \nabla s) = \hat{W}_e(E) + W_d(s) + W_i(\nabla s) \). Integrating \( W \) over the entire domain, we arrive at the energy functional of the system

\[ \Pi[s, \phi] = \int_{\Omega} \left[ -\frac{\epsilon(s)}{2} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i} + \frac{\Gamma}{l^2} \frac{1 - f(s)}{4} \frac{\partial s}{\partial x_i} \frac{\partial s}{\partial x_i} \right] dV. \]  

(7)

It is noteworthy that the formulation of the phase field model described here, especially the mathematical form of the complementary electrostatic energy function, is very similar to the phase field model of a conducting crack [27]. However, these two phenomena are very different physically. The conductive channel of dielectric breakdown, driven by the vector electric field, usually takes the one dimensional form. On the other hand, driven by the tensorial stress field, a crack most effectively reduces elastic energy as a two dimensional entity. This is because the electric field near the tip of a needle is much more concentrated than that near a sharp edge (e.g. Ref. [28]), while stress is more concentrated at the crack edge than at the end of a tunnel. The morphology of a conductive channel will further be verified numerically in Section IIIA.
B. Equilibrium state

Using variational calculus to minimize $\Pi$ with respect to $s$ and $\phi$, we can derive the Euler equations for an equilibrium state

$$\frac{\partial}{\partial x_i}\left[\epsilon(s)\frac{\partial \phi}{\partial x_i}\right] = 0,$$

(8)

$$\frac{\epsilon'(s)\frac{\partial \phi}{\partial x_i}}{2} + \frac{\Gamma'}{l^2} f'(s) + \frac{\Gamma \frac{\partial^2 s}{\partial x_i \partial x_j}}{2} = 0.$$

(9)

It could be recognized that Eq. (8) is Gauss’s law of electrostatics in the absence of distributed charge.

To study the energetics of the system, we consider the generalized energy momentum tensor

$$T_{ij} = W\delta_{ij} + \epsilon \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_j} - \frac{\Gamma}{2} \frac{\partial s}{\partial x_i} \frac{\partial s}{\partial x_j}.$$  

(10)

Using integration by parts and the divergence theorem, it could be shown that over any closed surface on a simply connected domain, the area integration of the normal projection of $T$ vanishes in equilibrium:

$$\oint_{\partial \Omega} T_{ij} n_i dA = 0,$$

(11)

where $n$ is the outward-pointing unit normal vector on the surface. In the intact material, a part of the integration recovers the path-independent $J$-integral for dielectric breakdown,

$$J = \oint_{\partial \Omega} \left( W_n n_i + n_j D_j E_i \right) dA \quad \text{[13].}$$

If the integration surface cut through the conductive channel, Eq. (11) suggests that the $J$-integral can be written as an integral only over a cross section $A_c$ of the conductive channel, where $s < 1$:

$$J = \int_{A_c} \left[ W_d + W_i - \frac{\Gamma}{2} \left( \frac{\partial s}{\partial x_i} \right)^2 \right] dA.$$  

(12)

Due to the arbitrariness of the integration surface, without losing generality, $A_c$ is selected to be normal to the conductive channel, i.e. the $x_i$-direction, and the outward normal is point away from the tip of the channel, $n_i = -1$. 

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To evaluate the integral, we consider a special case in which the conductive channel is straight and infinitely long. The dielectric medium is also assumed to be infinite. Such a case can be regarded as the approximation in the neighborhood of a conductive channel far behind its tip. The presence of the channel leads to a uniform electric potential, and the axisymmetry of the problem suggests that $s$ is only a function of the radial coordinate $r$. The stationary Euler equation (9) is thus reduced to an ordinary differential equation for $s(r)$:

$$f'(s) + \frac{l^2}{2} \frac{d^2 s}{dr^2} = 0.$$  

With boundary conditions $s(0) = 0$ and $s(\infty) = 1$, Eq. (13) can integrated to

$$\frac{ds}{dr} = \frac{2}{l} \sqrt{1 - f(s)}.$$  

(14)

The distribution of the damage field is obtained upon further integration and is plotted in Fig. 2. As expected, the degradation decays exponentially around the conductive channel, and is highly localized in a region of radius $\sim l$. With the aid of the solution in this special case, we can evaluate the $J$-integral. As $\partial s / \partial x_i = 0$, Eq. (12) is reduced to

$$J = 2\pi \frac{\Gamma}{l^2} \int_0^l \left[ 1 - f(s) + \frac{l^2}{4} \left( \frac{ds}{dr} \right)^2 \right] r dr = 2\pi \frac{\Gamma}{l} \int_0^l \sqrt{1 - f(s)} r ds.$$  

(15)

FIG. 2. Equilibrium distribution of the damage variable $s$ near a straight conductive channel at $r = 0$.

Substituting in the specific form of $f(s)$, and carrying out the integration numerically, we obtain $J \approx 0.982\Gamma$. Due to the reversible assumption of the damage field, the equilibrium state corresponds to the critical state for the growth of the conductive channel, when the driving force $J$ equals the resistance.
It is now clear that parameter $\Gamma$ is (approximately) the breakdown energy of the material. In the following discussion, we will neglect the small numerical difference, and regard $\Gamma$ as the breakdown energy.

It should be noted that the J-integral around a conductive channel in a linear dielectric material has a logarithmic dependence on the tip radius of curvature [12, 29], and a finite value may not be obtained around a mathematical line. This dependence may be removed by accounting for the leak current and charge injection in dielectrics [29]. Here this dependence is present, but the logarithmic singularity is avoided through the introduction of the length scale $l$. It is possible to extend the current model to include the effect of charge injection, although such an approach is beyond the scope of the current paper.

C. Equation of motion

Now let us consider the propagation of a conductive channel when the energetic driving force is supercritical. For simplicity, we will follow the phase-field model of brittle fracture [16, 22] and employ the linear kinetics for the damage process: $\frac{\partial s}{\partial t} = -m \frac{\partial \Pi}{\partial s}$, where $m$ is the mobility, a material parameter characterizing the speed of damage in the material. This model can always be extended by including nonlinear kinetics to better describe more complex processes. Substituting the energy functional to the kinetic equation, we arrive at

$$\frac{1}{m} \frac{\partial s}{\partial t} = \frac{\epsilon'(s)}{2} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i} + \frac{\Gamma}{2} \frac{f'(s)}{f(s)} + \frac{\Gamma}{2} \frac{\partial^2 s}{\partial x_i \partial x_i}.$$  \hspace{1cm} (16)

On the other hand, the partial electrostatic equilibrium, Eq. (8), is still insisted during a damage process. By solving the partial differential system (8) and (16) simultaneously with proper initial and boundary conditions, one can compute the coevolution of the electric field and material damage through $\phi(x,t)$ and $s(x,t)$.

To facilitate numerical calculation, in the following examples, we will normalize all lengths by $l$, energies by $\Gamma l$, time by $m \Gamma l$, and electric potentials by $\sqrt{\Gamma/\epsilon^0} l$. The resulting dimensionless governing equations read

$$\frac{\partial}{\partial x_i} \left[ \frac{1}{f(s) + \eta} \frac{\partial \phi}{\partial x_i} \right] = 0,$$  \hspace{1cm} (17)
\[
\frac{\partial s}{\partial t} = - \frac{f'(s)}{2[f(s) + \eta]} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i} + f'(s) + \frac{1}{2} \frac{\partial^2 s}{\partial x_i \partial x_i}.
\]

(18)

in which the symbols with over-bars are the dimensionless counterparts of the corresponding quantities.

III. RESULTS AND DISCUSSIONS

The partial differential equations are implemented into a finite element code through the commercial software, COMSOL Multiphysics 4.3, and used to solve the following initial boundary value problems as illustrative examples. In all examples, the electric potential field \( \phi(x, t) \) is interpolated with quadratic Lagrange elements, and the damage field \( s(x, t) \) is interpolated with linear Lagrange elements for compatibility.

A. Parallel capacitor with a field concentrator

![Figure 3](image)

FIG. 3. (a) Schematic of a cylindrical dielectric sample with a thin conductive field concentrator. Voltage is applied between the electrodes on the top and bottom surfaces. A conductive wire is placed on the axis and connected to the top electrode to act as the field concentrator. (b) Field-charge-density relation of the parallel capacitor before and after dielectric breakdown.

As a first example, we study the charging and breakdown process of a parallel capacitor with a circular cross-section. To concentrate the electric field and initiate breakdown in a controllable way, a thin conductor parallel to the field direction is placed in the middle of the capacitor, and connected to one
electrode, as sketched in Fig. 3a. The dimensionless radius of the dielectric medium is taken to be \( \bar{R} = 20 \) and the dimensionless thickness of the capacitor is \( \bar{H} = 20 \). This three dimensional problem is solved in an axisymmetric domain by assuming the breakdown path to always remain on the symmetry line. As electric boundary conditions, the potentials on the two electrodes as well as the field concentrator are given, and the lateral surface is left charge-free. For the damage variable \( s \), the natural boundary condition, a vanishing normal derivative \( n_i \partial s / \partial x_i \) is prescribed on all boundaries. Moreover, the value of \( s \) is fixed to be 0 on the field concentrator. At the initial time step, the iterative solver will find a consistent initial condition for \( s \) so that it changes continuously from 0 on the concentrator to 1 at the far field. The gradient energy term dictates the dimensionless thickness of the diffuse damage zone to be approximately 1. In fact, even though we have set the field concentrator to be a mathematical line, it always acts as a conductor with finite thickness by the nature of the phase-field method.

FIG. 4. Snapshots of the vertical cross-section of the axisymmetric model, showing the evolution of the conductive channel. The shading represents the damage field variable \( s \), with \( s = 1 \) indicating the intact state and \( s = 0 \) the fully damaged. The four snapshot, A, B, C, and D correspond to the four points on Fig. 3b.

Figure 3b plots the resulting relation between the nominal charge density on the bottom electrodes \( \bar{\sigma} \) and the nominal electric field \( \bar{E} = \bar{\phi} / \bar{H} \). The capacitor is loaded through a charge-controlled manner, with the total charge on one electrode ramping up linearly with time. The loading speed is set to be slow enough to represent the quasi-static behavior. Prior to the initiation of new damage zone in the dielectric medium, the electric field increases linearly with the charge density. By computing the J-integral numerically over the boundaries, we obtain the critical condition \( J_c / \Gamma \approx 1.2 \), and verify that the energy criterion for the propagation of the conductive channel has been executed. Once initiated, the conductive channel extends the preexisting field concentrator, leading to even stronger field concentration. The electric field needed for to further extend the channel thus decreases with its
extension. The propagation of the conductive channel is unstable. The propagation of the damaged conductive zone is shown by the snapshots of the field variable $s$ in Fig. 4.

The above presented calculation is carried out in an axisymmetric domain. To show that the model actually predicts a tubular channel, and the morphology is unaffected by the domain geometry and by the shape of the field concentrator, we also perform the calculation in a true 3D domain. The computational domain measures 20×20×10 in dimensionless units ($l$), with periodical boundary conditions applied in the lateral directions, and ramping voltage applied between the top and bottom surfaces. The voltage is increased quasi-statically to allow damage evolution. The field concentration is introduced geometrically by placing a shallow slit-like dent on the top surface, measured 4$l$ in length and 0.1$l$ in depth. As shown by Fig. 5, the spontaneously formed damage zone indeed takes a 1D tubular form along the direction of electric field. Despite the symmetric geometry of the domain, the calculated breakdown path is rather random, with the numerical source of asymmetry (e.g. through the random mesh). To ensure accuracy, a denser mesh is placed in the middle region where the channel will form. Such an approach may not be applicable in general, especially when the breakdown path is unknown. Therefore, in the examples of the following sections, the calculations are all done in 2D. More realistic 3D simulation may be done by large scale computing or with the aid of adaptive mesh regeneration.

![Conductive channel formed in a 3D domain.](image)

FIG. 5. Conductive channel formed in a 3D domain. The shading shows the damage variable $s$, with $s = 1$ indicating the intact state and $s = 0$ the fully damaged. The electric field is applied between the top and bottom surfaces, with a slit-like field concentrator on the top.

### B. Material with distributed defects

This example illustrates the usage of the model when a macroscopic field concentrator is not present. If we start from a perfectly homogeneous model, no localized damage will be obtained
numerically, and the entire domain will damage simultaneously. Instead, we build the numerical model with a number of randomly distributed defects. The defects are introduced by adding to the energy function a quadratic term in \( s \) with random coefficients, representing an energetically weaker material particle that favors the damaged phase. The random coefficients follow a normal distribution with zero mean value and a standard deviation of 0.3. A threshold is set so that more than half of the domain is defect free. In numerical calculations, these defects serve as the nuclei of the damaged phase. We compute the evolution of the material damage, and the results are shown as snapshots in Fig. 6. (Only the results after damage initiation are shown.) For the purpose of illustration, the calculation is done on a two-dimensional model. As expected, conductive channels initiate randomly in the material. When the sizes of these conductive channels are still sub-critical, several channels grow simultaneously. A catastrophic breakdown event takes place when a conductive channel beyond the critical length is formed via growth or percolation. Different from the case when a macroscopic field concentrator is present, here the conductive channel is not straight and contains branches.

![Fig. 6. Snapshots of the breakdown process of a defect-containing material. The shading represents the damage variable \( s \), with \( s = 1 \) representing the intact state and \( s = 0 \) the fully damaged. The damage zones nucleate near defects, grow, and percolate to form a conductive channel.](image)

Due to damage initiation and localization near defects, the nominal breakdown strength is much lower than the theoretical strength of the material. Without prior knowledge on the type and distribution of defects, we don’t expect our calculated critical field to match with experimental results on specific materials. However, the calculations do show the dependence of specimen sizes as observed in experiments [8, 30]. By keeping the size, distribution, and volume fraction of the defects to be the same while varying the height of the computational domain, we observe that the dielectric strength decreases as the sample size increases. As plotted in Fig. 7, the dielectric strength relates to the sample height approximately through an inverse power law, \( \bar{E} \propto H^{-\frac{1}{3}} \). Despite the coarse representation of defects, the results qualitatively agree with experimental observations [8, 30]. The quantitative difference may be
related to the dimension of the problem and the distribution of arbitrary defects. It should be noted that in the calculations, the defect size is kept constant by meshing the computational domains with elements of the same size, as the smallest defect is an element in finite-element calculation.

![Graph of dielectric strength vs. thickness](image)

**FIG. 7.** Calculated dependence of the dielectric strength on the sample thickness. The solid line, with a slope of -0.474, is the best fit to the numerical data points.

C. **Breakdown-resistant laminates**

Similar to the mechanical toughening effect of composites, dielectric solids can also be toughened electrically through a laminate structure. It has been proposed that by adding thin layers of conductive materials or weaker dielectrics to suppress field concentration and to dissipate energy, a homogeneous dielectric solid may be made more resistant to breakdown [12]. The effect is analogous to the blunting of a crack tip or the digression of a crack path. As the third example, we use our numerical model to study such an effect and demonstrate the usage of the model in designing dielectric composites.

As shown by Fig. 8, a two-dimensional model is constructed with two layers of materials with higher permittivity. The permittivity of these two layers is taken to be 100 times that in the bulk. To demonstrate the edge effect of the laminates, we introduced two gaps in the two thin layers of more permeable material. An electric field is added between the two electrodes on the upper and lower boundaries. A small conductive field concentrator is added to the upper boundary of the domain to initiate breakdown controllably. The side boundaries are left charge free. Defects are added in the same manner as in the previous example. Figure 8 shows the snapshots on the growth of the conductive channel during the breakdown process. Damage initiates from the tip of the field concentrator, and
propagates in the direction of the electric field, along which it reduces the most system potential energy. The damage propagation halts when it meets the layer of higher permittivity, which diffuses the field concentration from the tip of the channel. As shown by the field-charge plot in Fig. 9, the electric field carries on during the period when the conductive channel is arrested by the more permeable layer. The conductive channel is reinitiated at the corner of the more permeable layer, and propagates unstably until it reaches the next layer. As the laminate structure shown here contains only a few layers, the increase in the breakdown strength is not significant, but the energy dissipated through the breakdown process (the area below the field-charge curve) is much more than that of a homogeneous material (Fig. 3b). It should also be recognized that the intentionally added gaps in the permeable layers induces field concentration and lowers the breakdown strength. Even higher strength is expected if the gaps are reduced and the edges smoothed.

FIG. 8. Snapshots of the breakdown process of a dielectric laminate with thin layers of more permeable materials. The shading shows the damage variable \( s \), with \( s = 1 \) representing the intact state and \( s = 0 \) the fully damaged. Gaps are introduced intentionally in the more permeable layers to represent the worst-case scenario for damage re-initiation.
FIG. 9. The nominal field-charge relation of the laminate dielectric in Fig. 8. The points A through F on the plots correspond to the instances at which the snapshots in Fig. 8 are taken. The nominal electric field is calculated by dividing the applied voltage by the total thickness of the sample, and the nominal charge density is averaged over the entire bottom electrode.

IV. CONCLUSIONS

A phase-field model for the breakdown process of solid dielectrics is developed in this paper. The model tracks the initiation and growth of a conductive channel by evolving a continuous phase field of damage. Analogous to a crack in the phase-field models for fracture, the conductive channel is modeled by a thin tubular damage zone, separated from the background intact material by an interface with abrupt but continuous change in the phase field. To avoid direct handling of the intricate microscopic processes near the tip of a conductive channel, the model employs an energy criterion in analogy with that of crack propagation in linear elastic fracture mechanics: the electrostatic energy released equals (or exceeds) the energy consumption in growing the conductive channel by unit length. One-dimensional analysis of an axisymmetric case further shows that this criterion is equivalent to that given by the path-independent J-integral.

The model has been implemented numerically in the commercial finite-element package, COMSOL Multiphysics, and several examples are calculated for illustration. The calculation on a domain with randomly distributed defects recovers the inverse power-law relation between the breakdown strength and the thickness of the sample, which qualitatively agrees with experimental observations reported in the literature. The numerical method has also been used in simulating the breakdown process in a lamellar structure with alternating layers of dissimilar permittivity. The results show that such a structure effectively increases the energy dissipation and can be used for breakdown resistance.
By introducing more realistic electric-field-polarization relations, the model developed can be easily applied to the quantitative analysis of the performance of specific materials and composites against dielectric breakdown. Moreover, the model can be used to study the structural effect to breakdown in electric and electronic devices, and aid the designs against failure.

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REFERENCES